POLITECNICO DI MILANO

ECI 8th International Conference on Boiling and Condensation Heat Transfer
3-7 June 2012 Lausanne, Switzerland

Implementation of a Ghost Fluid Method in a Tree-Based Adaptive Volume of Fluid Solver for Two-Phase Heat and Mass Transfer

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Description of the problem

Several industrial applications make use of phase change phenomena in order to reach high heat transfer coefficients. New fields of application involving boiling or condensation are also continuously appearing due to this attractive characteristic. Accordingly, accurate predictive tools for the determination of the heat transfer coefficient and critical heat flux have been and are still of great interest.
Due to the numerical complexities linked with the solution of the governing equations of two-phase incompressible flow with phase change, developing a reliable numerical model for solving such problem has always been challenging.
In this study, the open source Gerris flow solver developed by Popinet (2003) is used. Recently, Popinet (2009) implemented a Volume of Fluid (VOF) technique using the analytical relations provided by Scardovelli and Zaleski (2000) for interface reconstruction on rectangular cells. This allowed the solution of the incompressible variable-density Navier-Stokes equations for two-phase flow with Adaptive Mesh Refinement (AMR) and second order global accuracy. In addition, Popinet (2009) implemented a generalized Height Function (HF) technique for the interface curvature calculation in order to obtain a second order accurate estimation of the interface curvature for the surface tension forces computation.
With the aim of providing a reliable model for solving two-phase incompressible flow with phase change, the mass conservation property offered by the VOF method together with the efficiency and accuracy given by the AMR and HF techniques, respectively, are coupled with the capabilities of the Ghost Fluid Method (GFM) for treating jump conditions across the interface as well as Dirichlet boundary conditions at the interface. As a preliminary step in the implementation, the performance given by the modified Poisson solver are shown by comparing analytical solutions with the numerical results for specific test cases.

Numerical approach

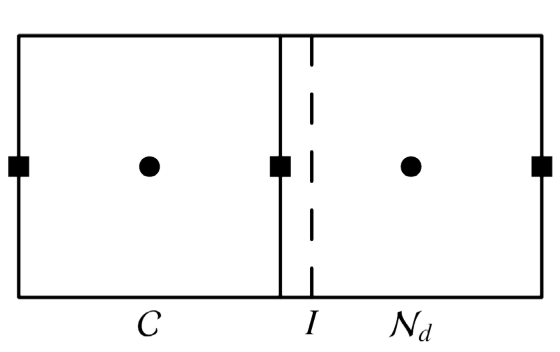
Details of the procedure for solving the variable-density incompressible Navier-Stokes equations as well as transport of additional scalars are given in Popinet (2003) and Popinet (2009).
In this preliminary implementation, the Gerris poisson solver is modified to implement interfacial jumps and Dirichlet interfacial boundary condition. The following variable coefficient Poisson equation is therefore considered

$$\nabla \cdot (\alpha \nabla u) = f$$

The implementation is based on the work of Liu et al. (2000) for jump conditions and on the work of Gibou et al. (2002) for Dirichlet boundary conditions at the interface. In general, it involves local modification of the Poisson coefficients and addition of constant terms to the right-hand-side of the discretized Poisson equation.

Conventions

An incompressible two-phase flow is considered and the interface separating the phases is identified as I. While discretizing at a cell face in direction d, the owner of the face is identified as C and the neighbor in direction d as Nd.



Interfacial jumps

The following interfacial jumps are considered in the solution of the Poisson equation

$$[u]_I = \Gamma_I = u^2 - u^1$$
$$[\alpha \nabla u \cdot \mathbf{n}]_I = [\alpha u_n]_I = \Lambda_I = \alpha^2 u_n^2 - \alpha^1 u_n^1$$

which can be interpolated from the cell center values as

$$\Gamma_I = \frac{\Gamma_C |\phi_{N_d}| + \Gamma_{N_d} |\phi_C|}{|\phi_C| + |\phi_{N_d}|}$$
$$\Lambda_I = \frac{\Lambda_C n_C^c |\phi_{N_d}| + \Lambda_{N_d} n_{N_d}^c |\phi_C|}{|\phi_C| + |\phi_{N_d}|}$$

with ϕ a level-set function.

Dirichlet interfacial boundary condition

The implementation requires the modification of the gradient computation to eventualy exclude neighbors. The interfacial boundary conditions, if not constant, can also be evaluated from cell center values as follow

$$u_I = \frac{u_{I,C} |\phi_{N_d}| + u_{I,N_d} |\phi_C|}{|\phi_C| + |\phi_{N_d}|}$$

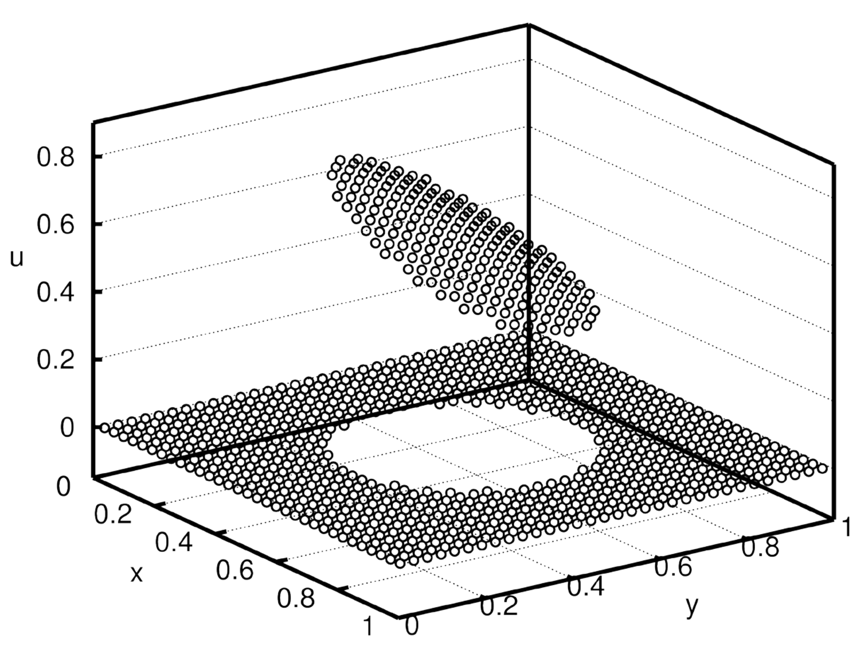
Results

In order to verify the proper implementation of the method, the examples presented in Liu et al. (2000) are reproduced and convergence analyses are performed. In particular, the accuracy of the reconstructed level set function from the VOF field is assessed by comparing results with an exact level set representation of the interface.

Interfacial jumps

The results show a global first order convergence of the GFM implemented in Gerris, which is expected considering the results obtained by Liu et al. (2000).

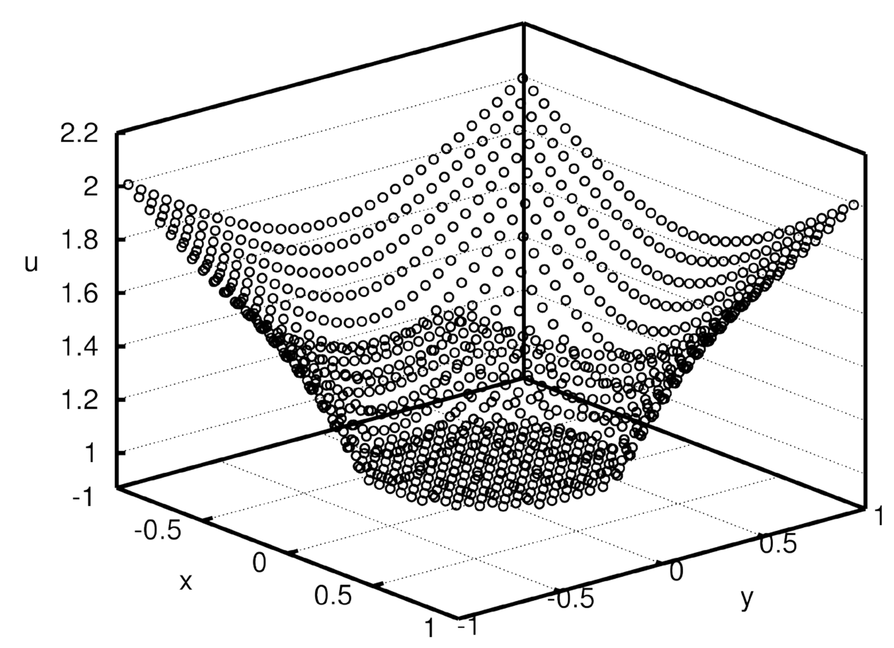
"example 3"



Grid	L_2 error in u	Order	L_∞ error in u	Order
8x8	0.004216		0.01246	
16x16	0.001244	1.76	0.00827	0.59
32x32	0.001114	0.16	0.00748	0.15
64x64	0.000369	1.60	0.00378	0.99
128x128	0.000142	1.37	0.00263	0.52
256x256	0.000079	0.84	0.00127	1.05

$$\nabla \cdot (\beta \nabla u) = f(x,y) \qquad \text{on } [0,1]x[0,1]$$
$$[u] = -e^{-x^2-y^2}$$
$$[\beta u_n] = 8(2x^2 + 2y^2 - x - y)e^{-x^2-y^2}$$

"example 5"



Grid	L_2 error in u	Order	L_∞ error in u	Order
8x8	0.009128		0.01913	
16x16	0.003152	1.53	0.00794	1.27
32x32	0.001536	1.04	0.00395	1.01
64x64	0.000565	1.44	0.00159	1.31
128x128	0.000237	1.25	0.00067	1.26
256x256	0.000083	1.51	0.00026	1.38

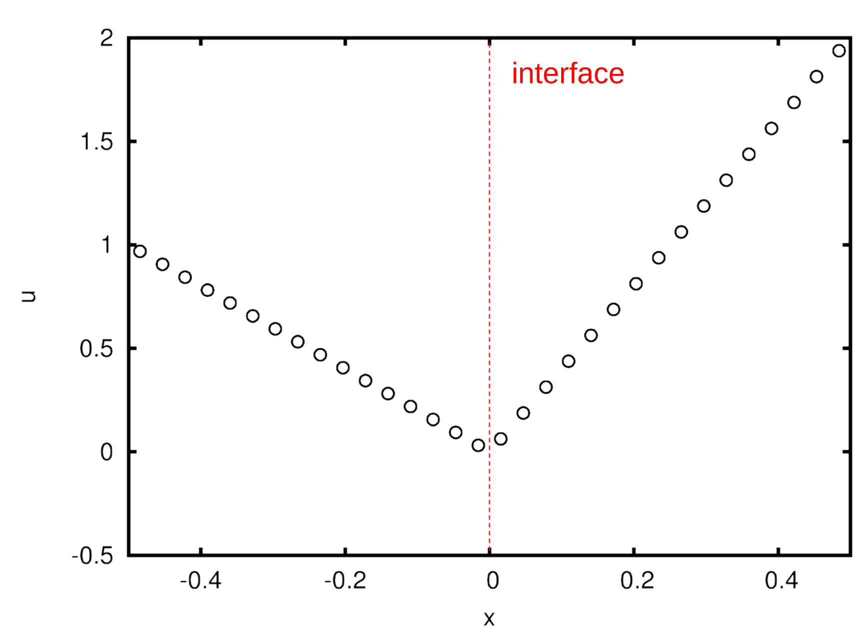
$$\Delta u = 0 \qquad \text{on } [-1,1]x[-1,1]$$
$$[u] = 0 \text{ and } [u_n] = 2$$

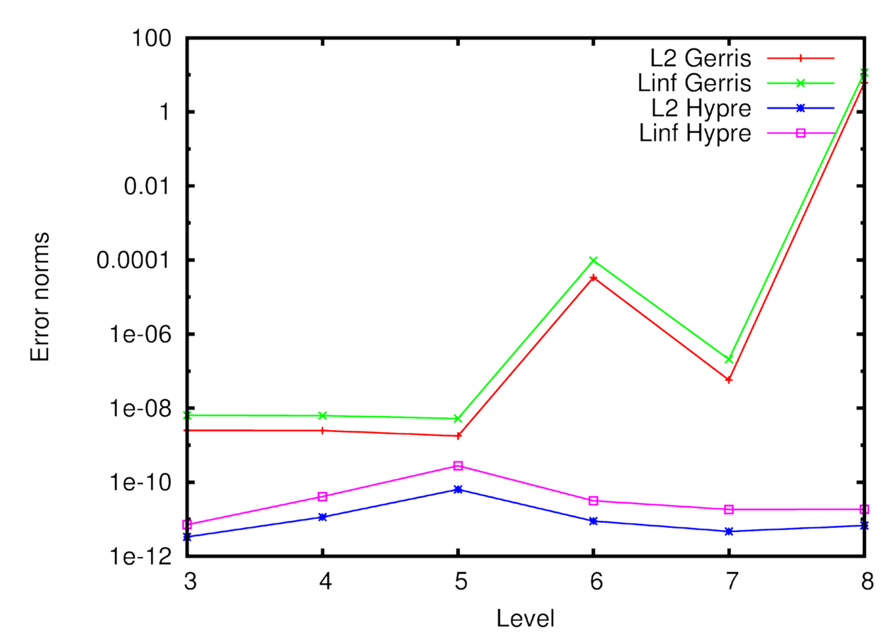
Level-set reconstruction

The differences in error norms for the "example 3" between a reconstructed level set from the VOF field and an exact level set representation of the interface are negligible. The use of a reconstructed level-set from the VOF field is thus validated.

Dirichlet interfacial boundary condition

A simple one-dimensional test case is considered. It is noticed that the Gerris native multigrid solver is not able to converge with grids superior to 32 cells. This is a significant issue that is solved using the hypre multigrid solver, available as an option in the code. Moreover for this simple case, the error norms are near machine accuracy and more complicated cases are required in order to assess the convergence rate of the method.





$$\Delta u = 0 \qquad \text{on } [-0.5,0.5] \qquad (u(-0.5) = 1) \qquad (u(0.5) = 2) \qquad (u(0) = 0)$$

Conclusions

A preliminary implementation of the Ghost Fluid Method within a tree-based adaptive Volume of Fluid solver has been performed. The resulting model combines the advantages of the Volume of Fluid interface tracking technique in terms of mass conservation, natural interface topology changes treatment and interface curvature calculation, and of the Ghost Fluid Method for treating jump conditions as well as Dirichlet boundary conditions at the interface. The results obtained from the model are compared with analytical solutions for specific test cases covering the Poisson solver with jump conditions and Dirichlet boundary conditions at the interface. A convergence study is also proposed in order to assess the validity and stability of the preliminary implementation and results are in good agreement with previous results in literature.

References

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